

# Isospin Dependence of $^1S_0$ Proton and Neutron Superfluidity in Asymmetric Nuclear Matter

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We investigate the  $^1S_0$  neutron and proton superfluidity in isospin asymmetric nuclear matter. We have concentrated on the isospin dependence of the pairing gaps and the effect of a microscopic three-body force. It is found that as the isospin asymmetry goes higher, the neutron  $^1S_0$  superfluid phase shrinks gradually to a smaller density domain, while the proton one extends rapidly to a much wider density domain. The three-body force turns out to weaken the neutron  $^1S_0$  superfluidity slightly, but it suppresses strongly the proton  $^1S_0$  superfluidity at high densities in nuclear matter with large isospin asymmetry.

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Superfluidity plays an important role in understanding a number of astrophysical phenomena in neutron stars [1, 2, 3, 4, 5, 6, 7, 8, 9]. In the inner crust of a neutron star where the total baryon density  $\rho$  is low, neutrons are thought to be superfluid in the  $^1S_0$  channel. In the nuclear core part (i.e., the outer core part), protons may form a superfluid in the  $^1S_0$  partial wave states, and neutron superfluidity is expected in the  $^3P_2$ - $^3F_2$  partial wave channel. It is generally expected that the cooling processes via neutrino emission [3, 4, 5], the properties of rotating dynamics, the post-glitch timing observations [6, 7], the possible vertex pinning [8] of a neutron star are rather sensitive to the presence of neutron and proton superfluid phases as well as to their pairing strength. Recently it has been shown [9] that a weak proton  $^1S_0$  superfluidity is in consistent with the cooling data if the effect of the possible presence of accreted envelopes is taken into account in the cooling scenario.

Nucleon superfluidity in symmetric nuclear matter and pure neutron matter has been investigated extensively by many authors using various theoretical approaches [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. All these investigations have predicted the occurrence of the  $^1S_0$  nucleon superfluid phase in the low density region although the obtained strengthes of the superfluidity are remarkably sensitive to the different approaches and the different approximations adopted. There also exists few investigations on the nucleon superfluidity in  $\beta$ -stable neutron star matter [25]. It is shown that due to the small proton fraction in  $\beta$ -stable matter, the proton superfluid phase in the  $^1S_0$  channel may extend to much higher baryon densities with a much smaller maximum of the pairing gap as compared to the case of symmetric nuclear matter. Therefore it is of interest to investigate

the variation of nucleon superfluidity vs. isospin asymmetry in asymmetric nuclear matter, which is expected to be helpful for understanding the properties of nucleon superfluid phases in neutron stars.

The aim of this work is devoted to the isospin dependence of the  $^1S_0$  neutron and proton superfluid phases in asymmetric nuclear matter, and to investigating the influence of three-body forces which turn out to be crucial for reproducing the empirical saturation properties of nuclear matter in a non-relativistic microscopic approach [26, 27, 28].

For such a purpose, we shall not go beyond the BCS framework. In this case, the pairing gap which characterizes the superfluidity in a homogeneous Fermi system is determined by the standard BCS gap equation [29], i.e.,

$$\Delta_{\vec{k}} = - \sum_{\vec{k}'} v(\vec{k}, \vec{k}') \frac{1}{2E_{\vec{k}'}} \Delta_{\vec{k}'}, \quad (1)$$

where  $v(\vec{k}, \vec{k}')$  is the realistic  $NN$  interaction in momentum space,  $E_{\vec{k}} = \sqrt{(\epsilon_{\vec{k}} - \epsilon_F)^2 + \Delta_{\vec{k}}^2}$ ,  $\epsilon_{\vec{k}}$  and  $\epsilon_F$  being the s.p. energy and its value at the Fermi surface, respectively. In the BCS gap equation, the most important ingredients are the realistic  $NN$  interaction  $v(\vec{k}, \vec{k}')$  and the neutron and proton s.p. energies  $\epsilon_{\vec{k}}$  in asymmetric nuclear matter. For the  $NN$  interaction, we adopt the Argonne V18 ( $AV_{18}$ ) two-body interaction [30] plus a microscopic three-body force (TBF). The TBF adopted in the present calculation was originally proposed in Ref. [26] based on the meson-exchange current approach. The parameters of the TBF, i.e., the coupling constants and the form factors, were determined [27] from the one-boson-exchange potential model to meet the self-consistent requirement with the adopted  $AV_{18}$  two-body force. A more detailed description of the TBF model and the related approximations can be found in Ref. [26].

The proton and neutron s.p. energies in asymmetric nuclear matter are calculated by using the BHF approach for isospin asymmetric nuclear matter [31]. The starting

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point of the BHF approach is the Brueckner  $G$ -matrix, which satisfies the Bethe-Goldstone (BG) equation [32]:

$$G(\rho, \beta; \omega) = v + v \sum_{k_1 k_2} \frac{|k_1 k_2 \rangle Q(k_1, k_2) \langle k_1 k_2|}{\omega - \epsilon(k_1) - \epsilon(k_2) + i\eta} G(\rho, \beta; \omega), \quad (2)$$

where  $k_i \equiv (\vec{k}_i, \sigma_i, \tau_i)$  denotes the momentum, the  $z$ -components of spin and isospin of a nucleon, respectively.  $\omega$  is the starting energy and  $Q(k_1, k_2)$  is the Pauli operator. The isospin asymmetry parameter is defined as  $\beta = (\rho_n - \rho_p)/\rho$ , being  $\rho_n$ ,  $\rho_p$ , and  $\rho$  the neutron, proton and total nucleon number densities, respectively. The s.p. energy is given by  $\epsilon(k) = \hbar^2 k^2/(2m) + U(k)$ . In solving the BG equation, we adopt the continuous choice [32] for the s.p. potential  $U(k)$  since it has been proved to provide a much faster convergency than the gap choice [33]. Under the continuous choice, the s.p. potential describes physically the nuclear mean field felt by a nucleon in nuclear medium [34] and is calculated from the real part of the on-shell  $G$ -matrix.

The neutron and proton superfluidity in nuclear matter can be described by their pairing energy gaps at their respective Fermi surfaces. To solve the gap equation, we follow the scheme given in Ref. [10]. The proton and neutron s.p. energies in the gap equation are calculated from the BHF approach. In our calculations, the TBF contribution has been included by reducing the TBF to an equivalently effective two-body interaction according to the standard scheme as described in Ref. [26]. A detailed description and justification of the method are discussed in Refs. [26, 27].

In Fig. 1 is shown the neutron energy gap in the  $^1S_0$  partial wave channel  $\Delta_F^n = \Delta(k_F^n)$  as a function of the total baryon density  $\rho$ . The curves along the direction of the arrow from the bottom correspond to  $\beta = 0.2, 0.4, 0.6$ , and  $0.8$ , respectively. We see that the neutron  $^1S_0$  superfluid phase exists only at low densities ( $\rho \leq 0.13 \text{ fm}^{-3}$ ) and the peaks of the pairing gaps are located around  $\rho = 0.02 \text{ fm}^{-3}$ , which is compatible with the previous predictions for pure neutron matter and symmetric nuclear matter [10, 24]. As the isospin asymmetry  $\beta$  increases, the neutron fraction increases for a given total density and as a consequence the density domain for the neutron  $^1S_0$  superfluidity shrinks gradually. In the case without including the TBF, the density region for the presence of the neutron superfluidity reduces from  $\rho \leq 0.13 \text{ fm}^{-3}$  down to  $\rho \leq 0.1 \text{ fm}^{-3}$  as the isospin asymmetry rises from  $\beta = 0.2$  to  $\beta = 0.8$ . In Ref. [24], we calculated the  $^1S_0$  pairing gaps in symmetric nuclear matter ( $\beta = 0$ ) and pure neutron matter ( $\beta = 1$ ). We found that the neutron  $^1S_0$  superfluid phase is expected to appear in a density range of  $\rho_B \leq 0.135 \text{ fm}^{-3}$  for symmetric nuclear matter and  $\rho_B \leq 0.09 \text{ fm}^{-3}$  for pure neutron matter, in consistent with our present results. The above obtained isospin dependence of the pairing gap is readily understood as follows. In asymmetric nu-

clear matter, the strength of the neutron pairing gap is related directly with the *neutron* number density. As the asymmetry rises, the neutron excess and the neutron number density for a given total nucleon density increase. As a consequences, the density domain for the existence of the neutron superfluid phase shrinks gradually going from symmetric matter to pure neutron matter. We notice that in both cases with and without including the TBF, the peak values of the gaps decrease and their location shift slightly to lower densities as the matter becomes more isospin asymmetric. This is mainly attributed to the isospin dependence of the neutron s.p. energy spectrum in asymmetric nuclear matter. As we know, the neutron s.p. potential in asymmetric matter becomes less deeper and the neutron effective mass gets larger for a larger isospin asymmetry according to the BHF and DBHF calculations [31, 35]. We may also see from Fig. 1 that within the region of the neutron superfluid phase, as the isospin asymmetry increases, the pairing gap becomes smaller at low densities below and in the vicinity of the peak position, while it gets larger at relatively high densities. The above obtained isospin behavior stems from the competition between two different mechanisms. On the one hand, at a fixed total density on the right side of the peak position, a higher asymmetry corresponds to a larger value of neutron density and a weaker neutron superfluidity. On the other hand, the isospin dependence of the neutron s.p. spectrum intends to enhance the neutron pairing gap as the isospin asymmetry increases. At low densities around and below the peak position, the later effect is dominated, while as the density increases, the former mechanism becomes more and more effective. Comparing the solid curves with the corresponding dashed ones, we may see that the TBF affects mainly the pairing gap at high densities. Inclusion of the TBF weakens the neutron superfluidity at high densities and makes the predicted density range for the existence of neutron superfluid phase smaller.

In Fig. 2 is reported the proton  $^1S_0$  pairing gap in asymmetric nuclear matter for four different asymmetries  $\beta = 0.2, 0.4, 0.6$ , and  $0.8$ , respectively. The solid curves are obtained by including the TBF, while the dashed ones without including the TBF. The isospin dependence of the proton  $^1S_0$  pairing gaps turns out to be completely different from that of the corresponding neutron ones in both cases with and without the TBF. It is seen that as the isospin asymmetry increases, the density domains for the existence of the proton superfluid phases enlarge rapidly, the peaks of the pairing gaps become lower appreciably and shift to higher densities gradually. As the asymmetry rises from  $\beta = 0.2$  to  $\beta = 0.8$ , the density domain extends from  $\rho \leq 0.17 \text{ fm}^{-3}$  to  $\rho \leq 0.435 \text{ fm}^{-3}$  in the case of not including the TBF and from  $\rho \leq 0.13 \text{ fm}^{-3}$  to  $\rho \leq 0.25 \text{ fm}^{-3}$  in the case of including the TBF, the peak values lower from about 2.0 MeV to about 1.4 MeV in both cases. The difference between the proton

and neutron superfluid phases is especially pronounced at high isospin asymmetries. As compared to the neutron  $^1S_0$  superfluidity, the proton  $^1S_0$  superfluid phase in highly asymmetric matter extends to much higher densities but with a smaller peak value of the pairing gap. For example, in the case without including the TBF, at  $\beta = 0.8$ , the proton  $^1S_0$  superfluidity is predicted to exist in the density region of  $\rho \leq 0.435 \text{ fm}^{-3}$  which is much wider than that of  $\rho \leq 0.1 \text{ fm}^{-3}$  for the neutron  $^1S_0$  superfluidity. The above behavior of the proton superfluidity vs. isospin asymmetry can be readily explained in terms of the isospin dependence of the proton density and the proton s.p. spectrum in asymmetric nuclear matter. First, at a fixed total density, a higher asymmetry corresponds to a smaller proton concentration and a lower proton density. Such an isospin dependence of the proton density is directly responsible for the widening of the density domain as a function of asymmetry. Second, the proton s.p. potential becomes deeper going from symmetric matter to pure neutron matter [31], and this results in the lowering of the proton pairing peak vs. asymmetry. At relatively low densities below and around the peak position, the variation of the proton s.p. spectrum as a function of asymmetry plays a major role in determining the isospin variation of the proton pairing gap and thus the proton superfluidity becomes weaker at a higher asymmetry. While as the density increases, the effect due to the decreasing of the proton fraction vs. asymmetry gets stronger. At high enough densities, it becomes predominant and leads to a turnover of the isospin behavior of the proton superfluidity (i.e., a stronger superfluidity at a higher asymmetry). The turnover is clearly seen in Fig. 2 and stems from the competition between the two above-mentioned isospin effects.

We notice that the TBF effect is negligibly small at low densities below and around the peak position. However, it gets stronger rapidly as the density goes up. The TBF turns out to induce a significant reduction of the proton pairing gaps in the high-density superfluidity domain. Consequently, it leads to a remarkable shrinking of the density domain for the existence of the proton superfluid phase. We notice that the above predicted TBF suppression of the  $^1S_0$  proton superfluidity is particularly pronounced for highly asymmetric matter. For example, at  $\beta = 0.8$ , the density domain of the proton superfluid phase is reduced by about 50%, i.e., from  $\rho \leq 0.435 \text{ fm}^{-3}$  down to  $\rho \leq 0.25 \text{ fm}^{-3}$ , by inclusion of the TBF. In nuclear matter, proton pairs are embedded in the medium of neutrons and protons, both the surrounded protons and neutrons contribute to the TBF renormalization of the proton-proton interaction, therefore the TBF effect on the proton pairing correlations is determined by the *total* nucleon number density instead of the *proton* number density. Accordingly, in spite of the small proton fractions and the low proton densities at high asymmetries, the TBF modifies strongly the proton-

proton pairing interactions at high-density superfluidity domain and weakens considerably the corresponding proton pairing gaps. One can verify readily from Fig. 2 that the TBF suppression of the proton superfluidity is mainly in the high-density region and the reduction of the gap increases rapidly as increasing the total density. We also notice from Fig. 2 that the TBF effect gets stronger as the asymmetry increases since the proton superfluid phase extends to larger densities for asymmetric matter at higher isospin asymmetries. Inclusion of the TBF weakens considerably the isospin dependence of the predicted proton superfluidity as compared to the results obtained by adopting purely the  $AV_{18}$  two-body force.

In summary, we have calculated the neutron and proton  $^1S_0$  pairing gaps in asymmetric nuclear matter based on the BHF approach and the BCS theory. We have especially investigated the isospin dependence of the pairing gaps in the  $^1S_0$  channel and the influence of the TBF. It is shown that the isospin dependence of the proton  $^1S_0$  superfluidity in asymmetric nuclear matter is completely different from that of the neutron one. The neutron  $^1S_0$  superfluid phase exists only in low density region for all isospin asymmetries. As the matter goes from symmetric nuclear matter to pure neutron matter, the peak value of the neutron  $^1S_0$  pairing gap becomes larger and the density domain for the existence of neutron superfluidity shrinks gradually. While the density domain for the proton superfluid phase enlarges rapidly as the isospin asymmetry rises and it may extend to very high densities for highly asymmetric nuclear matter. The peak value of the proton pairing gap turns out to be larger in asymmetric nuclear matter at a higher isospin asymmetry.

The TBF affects only weakly the neutron  $^1S_0$  superfluid phase in asymmetric nuclear matter, i.e., it reduces slightly the pairing gap, due to the low-density region for this kind of superfluidity. However it suppresses strongly the proton superfluidity in the  $^1S_0$  channel at high densities, especially at high asymmetries. The density domain for the existence of the proton  $^1S_0$  superfluid phase is reduced by about 50% from  $\rho \leq 0.435 \text{ fm}^{-3}$  to  $\rho \leq 0.25 \text{ fm}^{-3}$  by inclusion of the TBF.

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### Figure Captions:

FIG. 1: Neutron  $^1S_0$  energy gap as a function of density in asymmetric nuclear matter at various asymmetries. The solid curves are predicted by adopting purely the  $AV_{18}$  two-body interaction, while the dashed ones by using the  $AV_{18}$  plus the TBF.

FIG. 2: The same as Fig. 1 but for proton  $^1S_0$  pairing gap.



